Solving and using quadratic equations – Higher

Site :

Quadratic equations can be solved by factorising, completing the square and by formula.

Solving quadratic equations by factorising

To solve a quadratic equation, the first step is to write it in the form: $ax^2 + bx + c = 0$. Then factorise the equation as you have revised in the previous section.

If we have two numbers, A and B, and we know that A × B = 0, then it must follow that either A = 0, or B = 0 (or both). When we multiply any number by 0, we get 0.

Example

Solve the equation: $x^2 - 9x + 20 = 0$

Solution

- First, factorise the quadratic equation $x^2 - 9x + 20 = 0$
- Find two numbers which add up to 9 and multiply to give 20. These numbers are 4 and 5.

  $(x - 4)(x - 5) = 0$

- Now find the value x so that when these brackets are multiplied together the answer is 0.

  This means either $(x - 4) = 0$ or $(x - 5) = 0$

  So x = 4 or x = 5.

- You can check these answers by substituting 4 and 5 in to the equation:

  $x^2 - 9x + 20$

  Substituting 4 gives:

  $4^2 - 9 \times 4 + 20 = 16 - 36 + 20 = 0$

  Substituting 5 gives:

  $5^2 - 9 \times 5 + 20 = 25 - 45 + 20 = 0$

  Remember these 3 simple steps and you will be able to solve quadratic equations.

Now try this question.

Solving quadratic equations Solve $x^2 - 8x + 15 = 0$	

Completing the square

This is another way to solve a quadratic equation if the equation will not factorise.

It is often convenient to write an algebraic expression as a square plus another term. The other term is found by dividing the coefficient of x by 2, and squaring it.

Any quadratic equation can be rearranged so that it can be solved in this way.

Have a look at this example.

Example 1

Rewrite $x^2 + 6x$ as a square plus another term.

The coeffient of x is 6. Dividing 6 by 2 and squaring it gives 9.

$x^2 + 6x = (x^2 + 6x + 9) - 9$

$= (x + 3)^2 - 9$

Example 2

We have seen in the previous example that $x^2 + 6x = (x + 3)^2 - 9$

So work out $x^2 + 6x - 2$

$x^2 + 6x - 2 = (x^2 + 6x + 9) - 9 - 2 = (x + 3)^2 - 11$

Now try one for yourself.

Question

Solve $x^2 + 6x - 2 = 0$

REVEAL ANSWER

Example 3

Rewrite $2x^2 + 20x + 3$

Rewrite to get x^2 on its own.

$2(x^2 + 10x) + 3$

- The coefficient of x is 10. Divide 10 by 2, and square to get 25.
- $= 2 ((x + 5)^2 - 25) + 3$
- $= 2 (x + 5)^2 - 50 + 3$
- $= 2 (x + 5)^2 - 47$

Question

Now use the previous example to solve $2x^2 + 20x + 3 = 0$

REVEAL ANSWER

Using the quadratic formula

Using the technique of completing the square, you can make a formula which works for all quadratic equations.

- The most general way to write a quadratic equation is:
- $ax^2 + bx + c = 0$
- Here a, b and c are numbers that vary for different equations.
- So if the equation was:
- $2x^2 + 7x + 11 = 0$
- then a = 2, b = 7, c = 11.
  The formula for the solution is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This formula will work for all equations that can be solved.

Always try to factorise first. If the equation factorises, this is the easier method. In an exam, any question that asks for an answer to a quadratic equation correct to x decimal places should be solved using this formula. Take a look at this example:

- Solve $2x^2 - 5x - 6 = 0$
- Here a = 2, b = -5, c = -6
  Substituting these values in the formula, gives you:

$$X = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-6)}}{2(2)}$$

$$= \frac{5 \pm \sqrt{25 + 48}}{4}$$

$$= \frac{5 \pm \sqrt{73}}{4}$$

$$= \frac{5 \pm 8.544}{4}$$

So $= \frac{5 + 8.544}{4}$ or $= \frac{5 - 8.544}{4}$

$$= \frac{13.544}{4} \qquad\qquad = \frac{-3.544}{4}$$

$= 3.386 \qquad\qquad\quad = -0.886$

$= 3.39$ correct to 2dp $\qquad = -0.89$ correct to 2dp
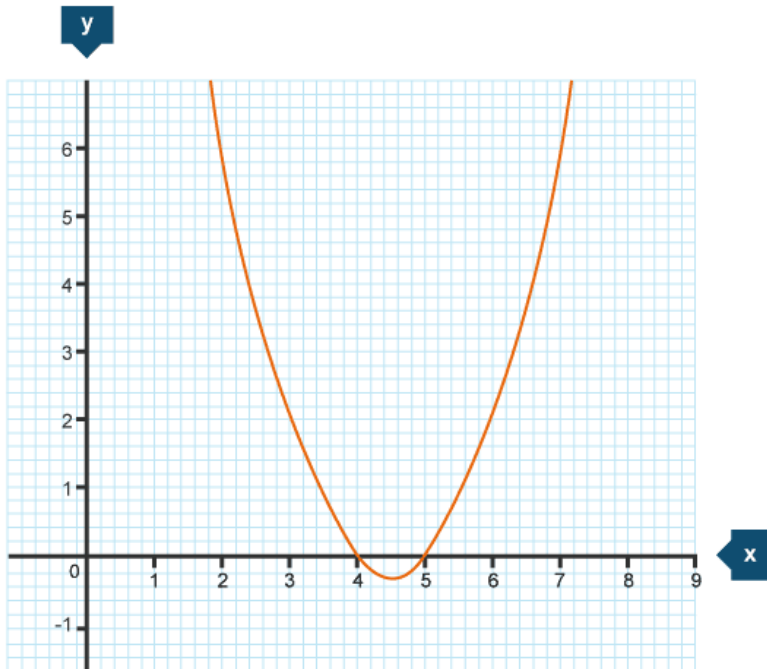
ⓐ Question

Now use the previous example to solve $2x^2 + 20x + 3 = 0$

[REVEAL ANSWER]

Solving a linear and quadratic simultaneous equation

Another way to solve a quadratic equation is to draw its graph.

This is the graph of: **$y = x^2 - 9x + 20$**

We can use it to solve: **$x^2 - 9x + 20 = 0$**

The answers are along the x axis where the graph reaches $y = 0$ (where it crosses the x axis). Using the graph the answers are $x = 4$ and $x = 5$. We can also solve this equation by factorising:
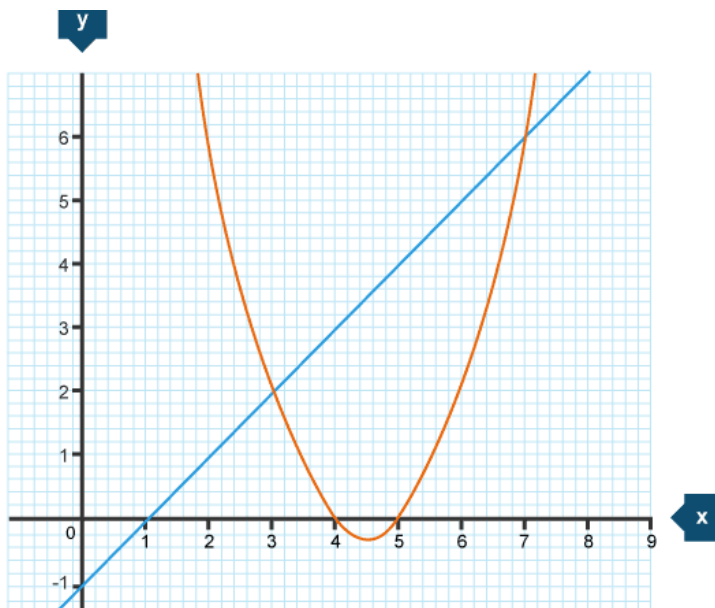
- $y = x^2 - 9x + 20$
- $y = (x - 4)(x - 5)$

This shows that the solutions are $x = 4$ and $x = 5$ which matches the answers in the graph above.

A graphical method can be used to find the solution of a quadratic and linear equation. Suppose we want to find the solutions for:

$x^2 - 9x + 20 = x - 1$

One side of this equation is $x^2 - 9x + 20$. We already know what this graph looks like.

The other side of the equation is $x - 1$. Drawing $y = x - 1$ on the same graph we get:

- The solutions to **x² - 9x + 20 = x - 1** can be found where the line and curve crosses.
- The solutions are found reading values from the x-axis.
- The solutions are x = 3 and x = 7.
We can check this with the following:

x = 3

1. 3^2 - 9 x 3 + 20 = 3 - 1
2. 9 - 27 + 20 = 2
3. 9 - 7 = 2
4. 2 = 2

x = 7

1. 7^2 - 9 x 7 + 20 = 7 - 1
2. 49 - 63 + 20 = 6
3. 49 - 43 = 6
4. 6 = 6

For the following questions, you may find it useful to look back at the graph sections **Straight-line graphs** and **Curved graphs**.

ⓐ Question

Solve: $x^2 + x + 2 = 5 - x$

REVEAL ANSWER

ⓐ Question

Solve: $x^2 + 2x + 1 = x + 4$

REVEAL ANSWER