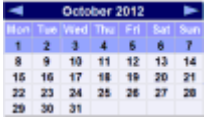# Probability: Complement

Complement of an Event: All outcomes that are **NOT** the event.

When the event is **Heads**, the complement is **Tails**

When the event is **{Monday, Wednesday}** the complement is **{Tuesday, Thursday, Friday, Saturday, Sunday}**

When the event is **{Hearts}** the complement is **{Spades, Clubs, Diamonds, Jokers}**

So the Complement of an event is all the **other** outcomes (**not** the ones we want). And together the Event and its Complement make all possible outcomes.

Probability

Probability of an event happening = *Number of ways it can happen* **Total number of outcomes**

Example: the chances of rolling a "4" with a die

Number of ways it can happen: 1 (there is only 1 face with a "4" on it)

Total number of outcomes: 6 (there are 6 faces altogether)

So the probability = *1*6
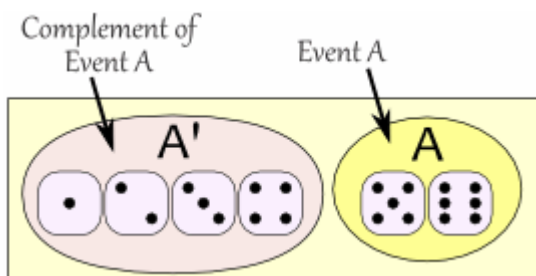
The probability of an event is shown using "P":

P(A) means "Probability of Event A"

The complement is shown by a little mark after the letter such as A' (or sometimes A^c or A):

P(A') means "Probability of the complement of Event A"

The two probabilities always add to 1

P(A) + P(A') = 1

Example: Rolling a "5" or "6"

Event A is {5, 6}

Number of ways it can happen: 2

Total number of outcomes: 6

P(A) = *2*6 = *1*3

The **Complement of Event A** is {1, 2, 3, 4}

Number of ways it can happen: 4

Total number of outcomes: 6

P(A) = $\frac{4}{6}$ = $\frac{2}{3}$

Let us add them:

P(A) + P(A') = $\frac{1}{3}$ + $\frac{2}{3}$ = $\frac{3}{3}$ = 1

Yep, that makes 1

It makes sense, right? **Event A** plus all outcomes that are **not Event A** make up all possible outcomes.

Why is the Complement Useful?

It is sometimes easier to work out the complement first.

Example. Throw two dice. What is the probability the two scores are different?

Different scores are like getting a **2 and 3**, or a **6 and 1**. It is quite a long list:

A = { (1,2), (1,3), (1,4), (1,5), (1,6),
(2,1), (2,3), (2,4), ... etc ! }

But the complement (which is when the two scores are the same) is only **6 outcomes**:

A' = { (1,1), (2,2), (3,3), (4,4), (5,5), (6,6) }

And the probability is easy to work out:

P(A') = 6/36 = **1/6**

Knowing that P(A) and P(A') together make 1, we can calculate:

P(A) = 1 − P(A')
   = 1 − 1/6
= **5/6**

So in this case (and many others) it's easier to work out P(A') first, then find P(A)