Hunger is no game

The Great Famine in Ireland

Problem statement

Great Famine, also called Irish Potato Famine, or Great Irish Famine that occurred in Ireland in 1844-1849 when the potato crop failed in successive years. The crop failures were caused by late blight, a disease that destroys both the leaves and the edible roots, or tubers of the potato plant. The causative agent of late blight is the water mold *Phytophthora infestans*. The Irish Famine was the worst to occur in Europe in the 19th century.

In 1844, the population was 8.2 million people in Ireland and at the same time, the potatoes production was 15 000 in thousands of tons.
We suppose that the population grows by 1.6 % per year and the potatoes production decreases by 0.5 thousand tons each year.

Questions

Question 1)
   a) Give the expression of the sequence related to the population evolution (U_n) and the sequence linked to the potatoes production (V_n).

   b) What is the nature of each sequence?

Question 2)
Determine the variations of both sequences.

Question 3)
Determine when the potatoes production won't be able to feed de Irish population? Justify.

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CORRECTION

Question 1)

a)

$U_n = 8.2 \times 1.016^n$

$V_n = 15 - n \times 0.5$

b)

The nature of (U_n)

$U_{n+1} = 8.2 \times 1.016^{n+1}$

$= 8.2 \times 1.016 \times 1.016$

$U_{n+1} = (U_n) \times 1.016$

(U_n) is a geometric sequence with common ratio equal to 1.016 and first term U_0 equal to 8.2.

The nature of (V_n)

$V_{n+1} = 15 - (n+1) \times 0.5$

$= 15 - n \times 0.5$

$= 15 - n \times 0.5 - 1$

$V_{n+1} = (V_n) - 1$

(V_n) is an arithmetic sequence with common difference equal to -1 and first term V_0 equal to 15.

Question 2)

Variations of (U_n)

(U_n) is in the form of **$U_n = U_0 \times q^n$** where **q** is the common ration, equal to 1.016

==> $U_n = 8.2 \times 1.016^n$

- ➢ **U_0 = 8.2 > 0**
- ➢ q = 1.016 > 1

==> (U_n) is increasing.

Variations of (V_n)

(V_n) is in the form of **$V_n = V_0 + n \times r$** where **r** is the common difference, equal to -1

- ➢ r = -1 < 0

==> (V_n) is decreasing.

Question 3)

(U$_n$) is a geometric sequence

(V$_n$) is an arithmetic sequence

(V$_n$) > (U$_n$)

==> $15 - n \times 0.5 > 8.2 \times 1.016^n$

With the calculator, we associate each sequence to a function and with the graph, we see the intersection of the curves.

Finally, we find that the potatoes production won't be able to feed the Irish population after 11 years.